Code No. **041**

Roll No. | | | | | | | |

- Please check that this question paper contains **4** printed pages.
- Code number given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.
- Please check that this question paper contains **33** questions.
- **Please write down the Serial Number of the question before attempting it.**
- 15 minutes time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the students will read the question paper only and will not write any answer on the answer-book during this period.

MATHEMATICS–XII
Sample Paper (Solved)

Time allowed: 3 hours *Maximum Marks: 80*

General Instructions:

Same as CBSE Sample Question Paper.

PART A

Section I

All questions are compulsory. In case of internal choices attempt anyone.

1. Let A = {1, 2, 3}, B = {4, 5, 6} and let f = {(1, 4), (2, 5), (3, 6)} be a function from A to B. State whether f is one-one or not.

Or

If R = $\{(x, y) : x + 2y = 8\}$ is a relation on N, write the range of R.

2. What are the possible number of relations on A = {1, 2, 3}.

3. Using the principal values, evaluate: $\tan^{-1} 1 + \sin^{-1}\left(-\frac{1}{2}\right)$.

Or

Using principal values evaluate: $\cos^{-1}\left(\cos\frac{2\pi}{3}\right) + \sin^{-1}\left(\frac{\sin 2\pi}{3}\right)$.

4. If A is a matrix of order 2 × 3 and B is a matrix of order 3 × 5, what is the order of matrix (AB)′ or T?

5. If A is a square matrix of order 3 such that $|adj\ A| = 64$, find $|A'|$.

Or

If A is a square matrix satisfying $A^2 = I$, then what is the inverse of A?

6. If $\Delta = \begin{vmatrix} 1 & 3 & -2 \\ 4 & -5 & 6 \\ 3 & 5 & 2 \end{vmatrix}$, Write the cofactor of a_{32} (the element of third row and second column).

7. Find $\int \frac{3 + 3\cos x}{x + \sin x}\, dx$.

Or

Find $\int(\cos^2 2x - \sin^2 2x)dx$

8. Find the integrating factor of the differential equation $\left(\frac{e^{-2\sqrt{x}}}{\sqrt{x}} - \frac{y}{\sqrt{x}}\right)\frac{dy}{dx} = 1.$

9. Write the order and degree of the differential equation $y = x\frac{dy}{dx} + a\sqrt{1+\left(\frac{dy}{dx}\right)^2}$.

Or

Find the integral factor for $(1 + x^2)\frac{dy}{dx} + 2xy - 4x^2 = 0.$

10. If $\vec{a}$ and $\vec{b}$ are two non-zero vectors such that $|\vec{a} \times \vec{b}| = \vec{a}\,.\,\vec{b}$, then find the angle between $\vec{a}$ and $\vec{b}$.

11. If $|\vec{a} + \vec{b}| = 60$, $|\vec{a} - \vec{b}| = 40$ and $|\vec{a}| = 22$, then find $|\vec{b}|$.

12. Give an example of vectors $\vec{a}$ and $\vec{b}$ such that $|\vec{a}| = |\vec{b}|$ but $\vec{a} \neq \vec{b}$.

13. Find the acute angle which the line with direction cosines $\frac{1}{\sqrt{3}}, \frac{1}{6}, n$ makes with positive direction of z-axis.

14. Find the direction cosines of the line: $\frac{x-1}{2} = -y = \frac{z+1}{2}$.

15. If A and B are two independent events, prove that A′ and B are also independent.

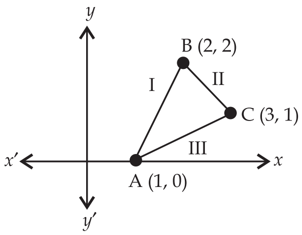
16. One bag contains 3 red and 5 black balls. Another bag contains 6 red and 4 black balls. A ball is transferred from first bag to the second bag and then a ball is drawn from the second bag. Find the probability that the ball drawn is red.

Section II

Both the case-study based questions are compulsory. Attempt any 4 sub parts from each question (17–21) and (22 – 26). Each question carries 1 mark.

17. Case Study—*Triangular Sandwich*

Top view of Sandwich



Answer the following the questions:

(*i*) Write the equation for line I in the form of *x*.

(*a*) $y = 2x + 1$ (*b*) $y = 2(x - 1)$ (*c*) $y = 2x + 2$ (*d*) $y = 3x + 1$

(*ii*) Write the equation for line II in the form of *x*.

(*a*) $y = 5 - x$ (*b*) $y = 3 - x$ (*c*) $y = 4 - x$ (*d*) $y = -x - 4$

(*iii*) Write the equation for line III in the form of *x*.

(*a*) $y = \frac{x+1}{3}$ (*b*) $y = x - 2$ (*c*) $y = \frac{x+1}{2}$ (*d*) $y = \frac{x-1}{2}$

(*iv*) Using integration, find the area of ΔABC.

(*a*) $\frac{3}{2}$ (*b*) $\frac{1}{2}$ (*c*) $\frac{3}{4}$ (*d*) $\frac{2}{3}$

(*v*) The relation between the lines are :

(*a*) parallel (*b*) perpendicular (*c*) Intersecting (*d*) None of the above

18. Case Study—A shopkeeper sells three types of flower seeds A_1, A_2, A_3. They are sold as a mixture where the proportions are 4 : 4 : 2 respectively. The germination rate of three types of seeds are 45%, 60% and 35%.

Answer the following questions:

(*i*) The probability of random chosen seed to germinate:

(*a*) .69 (*b*) .39 (*c*) .49 (*d*) .59

(*ii*) The probability that seed will not germinate, given that seed is of type A_3

(*a*) $\frac{15}{100}$ (*b*) $\frac{65}{100}$ (*c*) $\frac{75}{100}$ (*d*) $\frac{55}{100}$

(*iii*) The probability that the seed is of type A_2 given that randomly chosen seed does not germinate.

(*a*) $\frac{22}{51}$ (*b*) $\frac{16}{51}$ (*c*) $\frac{20}{51}$ (*d*) $\frac{10}{51}$

(*iv*) Calculate the probability that it is of type A_1, given that randomly chosen seed does not germinate

(*a*) $\frac{51}{22}$ (*b*) $\frac{22}{51}$ (*c*) $\frac{16}{51}$ (*d*) $\frac{1}{51}$

(*v*) The probability that it will not germinate given that seed of type A_1:

(*a*) $\frac{55}{100}$ (*b*) $\frac{65}{100}$ (*c*) $\frac{35}{100}$ (*d*) $\frac{45}{100}$

PART B

Section III

19. Prove the following : $\cos[\tan^{-1}\{\sin(\cot^{-1} x)\}] = \sqrt{\frac{1+x^2}{2+x^2}}$.

20. Find the inverse of the matrix $\begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix}$. Hence, find the matrix P satisfying the matrix equation $P\begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$.

Or

If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, find k so that $A^2 = 5A + kI$.

21. If $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$, then prove that $\frac{dy}{dx} = \frac{\sqrt{1-y^2}}{\sqrt{1-x^2}}$.

22. Find the intervals in which the function $f(x) = 3x^4 - 4x^3 - 12x^2 + 5$ is

(*a*) strictly increasing (*b*) strictly decreasing

23. Evaluate : $\int \frac{1+\sin 2x}{1+\cos 2x} \cdot e^{2x}\, dx$.

Or

Evaluate: $\int \frac{\sin\phi}{\sqrt{\sin^2\phi + 2\cos\phi + 3}}\, d\phi$.

24. Using integration, find the area of the region bounded by the curve $x^2 = 4y$ and the line $x = 4y - 2$.

25. Find the shortest distance between the lines :

$$\vec{r} = 6\hat{i} + 2\hat{j} + 2\hat{k} + \lambda(\hat{i} - 2\hat{j} + 2\hat{k}) \text{ and } \vec{r} = -4\hat{i} - \hat{k} + \mu(3\hat{i} - 2\hat{j} - 2\hat{k}).$$

26. If the sum of two unit vectors is a unit vector, show that the magnitude of their difference is $\sqrt{3}$.

27. Find the equation of the line which intersects the lines $\frac{x+2}{1} = \frac{y-3}{2} = \frac{z+1}{4}$ and $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ passes through the point (1, 1, 1).

28. Four bad oranges are accidentally mixed with 16 good ones. Find the probability distribution of the number of bad oranges when two oranges are drawn at random from this lot. Find the plan and variance of the distribution.

Or

A man is known to speak the truth 3 out of 4 times. He throws a die and reports that it is a six. Find the probability that it is actually a six.

Section IV

All questions are compulsory. In case of internal choices attempt any one.

29. Check whether the relation R in the set R of real numbers, defined by $R = \{(a, b) : 1 + ab > 0\}$, is reflexive, symmetric or transitive.

30. If the following function is differentiable at $x = 2$, then find the values of a and b, $f(x) = \begin{cases} x^2, & \text{if } x \le 2 \\ ax + b, & \text{if } x > 2 \end{cases}$

31. Let $y = (\log x)^x + x^{x \cos x}$, then find $\frac{dy}{dx}$.

Or

If $x = a \sin pt$, $y = b \cos pt$, then find $\frac{d^2y}{dx^2}$ at $t = 0$.

32. Find the equation of the normal to the curve $2y = x^2$, which passes through the point (2, 1).

33. Evaluate the following : $\int_0^{\frac{\pi}{2}} \frac{x \sin x \cos x}{\sin^4 x + \cos^4 x} dx$.

34. Compute, using integration, the area bounded by the lines $x + 2y = 2$, $y - x = 1$ and $2x + y = 7$

Or

Using the method of integration, find the area of the region bounded by the lines $2x + y = 4$, $3x - 2y = 6$ and $x - 3y + 5 = 0$.

35. Solve the following differential equation, $(1 + y + x^2y)dx + (x + x^3)dy = 0$, where $y = 0$ when $x = 1$.

Section V

All questions are compulsory. In case of internal choices attempt any one.

36. Use product $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix} \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$ to solve the system of equations

$x + 3z = 9, -x + 2y - 2z = 4, \quad 2x - 3y + 4z = -3.$

Or

Use product $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix} \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$ to solve the system of equations :

$x - y + 2z = 1; \qquad 2y - 3z = 1; \qquad 3x - 2y + 4z = 2.$

37. Find the equation of the plane passing through the point (1, 1, 1) and containing the line $\vec{r} = (-3\hat{i} + \hat{j} + 5\hat{k}) + \lambda(3\hat{i} - \hat{j} - 5\hat{k})$.

Also, show that the plane contains the line $\vec{r} = (-\hat{i} + 2\hat{j} + 5\hat{k}) + \lambda(\hat{i} - 2\hat{j} - 5\hat{k})$

Or

Show that the lines $\vec{r} = (-3\hat{i} + \hat{j} + 5\hat{k}) + \lambda(-3\hat{i} + \hat{j} + 5\hat{k})$ and $\vec{r} = (-\hat{i} + 2\hat{j} + 5\hat{k}) + \mu(-\hat{i} + 2\hat{j} + 5\hat{k})$ are coplanar. Also, find the equation of the plane containing these lines.

38. Solve the following graphically and also find the maximum profit.

Maximum Profit, $Z = 8000x + 12000y$

Subject to the constraints: $9x + 12y \le 180$; $3x + 4y \le 60$; $x \ge 0, y \ge 0$.

Or

Solve the following graphically and also find the maximum profit.

Maximum Profit, $Z = 6x + 3y$

Subject to the constraints: $4x + y \ge 80$; $x + 5y \ge 0$; $3x + 2y \le 150$; $x \ge 0, y \ge 0$.

Answer Sheet

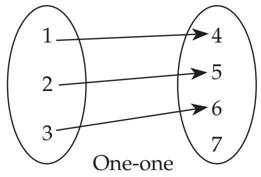
S	A	M	P	L	E							P	A	P	E	R		

Code No. 041

Roll No. | | | | | | | |

MATHEMATICS

1. A = {1, 2, 3}, B = {4, 5, 6, 7}
$f = \{(1, 4), (2, 5), (3, 6)\}$
$f(1) = 4, f(2) = 5, f(3) = 6$
Here different elements of A have different images in B.
So, ***f* is a one-one function**.



Or

We have, $x + 2y = 8 \quad \Rightarrow 2y = 8 - x \quad \Rightarrow y = \frac{8-x}{2}$

when $x = 1, y = \frac{7}{2} \notin N$ when $x = 2, y = 3 \in N$ when $x = 3, y = \frac{5}{2} \notin N$

when $x = 4, y = 2 \in N$ when $x = 5, y = \frac{3}{2} \notin N$ when $x = 6, y = 1 \in N$

when $x = 7, y = \frac{1}{2} \notin N$ when $x = 8, y = 0 \notin N$

$\therefore$ **Range = {1, 2, 3}**

2. Number of elements in A = 3 = p(let)
Number of elements in A × A = $p^2 = 3 \times 3 = 9$
$\therefore$ Possible number of relations on A = $2^{p^2} = \mathbf{2^9}$

3. $$\tan^{-1}(1) + \sin^{-1}\left(-\frac{1}{2}\right) = \tan^{-1}\left(\tan\frac{\pi}{4}\right) - \sin^{-1}\left(\frac{1}{2}\right)$$ **[$\because \sin^{-1}(-\theta) = -\sin^{-1}\theta$]**

$$= \frac{\pi}{4} - \sin^{-1}\left(\sin\frac{\pi}{6}\right) = \frac{\pi}{4} - \frac{\pi}{6} = \frac{3\pi - 2\pi}{12} = \mathbf{\frac{\pi}{12}}$$

Or

$$\cos^{-1}\left(\cos\frac{2\pi}{3}\right) + \sin^{-1}\left(\frac{\sin 2\pi}{3}\right) = \frac{2\pi}{3} + \sin^{-1}\left[\sin\left(\frac{3\pi - \pi}{3}\right)\right]$$

$$= \frac{2\pi}{3} + \sin^{-1}\left[\sin\left(\pi - \frac{\pi}{3}\right)\right]$$

$$= \frac{2\pi}{3} + \sin^{-1}\left(\sin\frac{\pi}{3}\right)$$ **...[$\because \sin(\pi - \theta) = \sin\theta$]**

$$= \frac{2\pi}{3} + \frac{\pi}{3} = \frac{3\pi}{3} = \pi$$

4. Order of matrix A = 2×3

Order of matrix B = 3×5

$\therefore$ Order of matrix AB = 2×5

$\therefore$ Order of matrix $(AB)^T$ or $(AB)'$ = **5 × 2**

5. $|Adj\ A| = |A|^{3-1}$ $\quad$ $[\because |Adj\ A| = |A|^{n-1}]$

$64 = |A|^2$ $\quad \therefore |A| = \pm 8$

$\because |A'| = |A|$ $\quad \therefore$ $\mathbf{|A'| = \pm 8}$

Or

$A^2 = I$

$AA = I$

$A^{-1}(AA) = A^{-1} I$ $\quad$ **(Pre multiplying by A^{-1})**

$(A^{-1}A)A = A^{-1}$ $\quad$ $[\because A^{-1} I = A^{-1}$

$IA = A^{-1}$ $\quad$ $[\because A^{-1} A = I$

$\therefore$ $\mathbf{A = A^{-1}}$ $\quad$ $[\because IA = A = AI$

Hence the inverse of A is A^{-1}.

6. The cofactor of $a_{32} = (-1)^{3+2}\begin{vmatrix} 1 & -2 \\ 4 & 6 \end{vmatrix} = -(6+8) = \mathbf{-14}$

7. We have $\int \frac{3 + 3\cos x}{x + \sin x} dx = 3\int \frac{(1+\cos x)}{x+\sin x} dx$ $\quad$ $\ldots\left[\begin{array}{l}\text{Let } p = x = \sin x \\ dp = (1+\cos x)\,dx\end{array}\right.$

$$= 3\int \frac{dp}{p} = 3\log|p| + c = \mathbf{3\log|x + \sin x| + c}$$

Or

$\int (\cos^2 2x - \sin^2 2x) dx$

$= \int \cos 2(2x) dx = \int (\cos 4x) dx = \frac{1}{4}\mathbf{\sin 4x + c}$ $\quad$ $\ldots[\because \cos^2\theta - \sin^2\theta = \cos 2\theta$

8. $\left(\frac{e^{-2\sqrt{x}}}{\sqrt{x}} - \frac{y}{\sqrt{x}}\right)\frac{dx}{dy} = 1$

$\Rightarrow \frac{e^{-2\sqrt{x}}}{\sqrt{x}} - \frac{y}{\sqrt{x}} = \frac{dy}{dx}$ $\qquad \Rightarrow \frac{dy}{dx} + \frac{y}{\sqrt{x}} = \frac{e^{-2\sqrt{x}}}{\sqrt{x}}$

Here 'P' $= \frac{1}{\sqrt{x}} = x^{-1/2}$

$\therefore$ **Integrating Factor** $= e^{\int P\,dx} = e^{\int x^{-1/2} dx} = \mathbf{e^{2x^{1/2}}}$ or $\mathbf{e^{2\sqrt{x}}}$

9. $y = x\frac{dy}{dx} + a\sqrt{1 + \left(\frac{dy}{dx}\right)^2}$

$\Rightarrow \left(y - x\frac{dy}{dx}\right)^2 = a^2\left[1 + \left(\frac{dy}{dx}\right)^2\right]$ $\quad$ **...[Squaring both the sides**

$\Rightarrow (x^2 - a^2)\left(\frac{dy}{dx}\right)^2 - 2xy\left(\frac{dy}{dx}\right) + (y^2 - a^2) = 0.$

In this equation, the order of the highest order derivative is **1** and its power is **2**. So, the order is 1 and the degree is 2 of the given differential equation.

Or

$(1 + x^2)\frac{dy}{dx} + 2xy - 4x^2 = 0$

Dividing both sides by $(1 + x^2)$, we have $\frac{dy}{dx} + \frac{2x}{1+x^2}y = \frac{4x^2}{1+x^2}$

Comparing the above with $\frac{dy}{dx} + Py = Q$, we have 'P' $= \frac{2x}{1+x^2}$, 'Q' $= \frac{4x^2}{1+x^2}$

$\therefore$ **Integral Factor** = I.F. = $e^{\int P\,dx} = e^{\int \frac{2x}{1+x^2}dx}$

$= e^{\int \frac{dt}{t}} = e^{\log|t|} = t = \mathbf{1 + x^2}$ ...**[Let $t = 1 + x^2 \Rightarrow dt = 2x\,dx$ and $e^{\log A} = A$**

10. Let θ be the angle between $\vec{a}$ and $\vec{b}$.

Given. $|\vec{a} \times \vec{b}| = \vec{a}\,.\,\vec{b}$ ...(*i*)

$|\vec{a}||\vec{b}| \sin\theta = |\vec{a}||\vec{b}| \cos\theta$

$\Rightarrow \sin\theta = \cos\theta \quad \Rightarrow \frac{\sin\theta}{\cos\theta} = 1$

$\Rightarrow \tan\theta = 1 \quad \therefore \theta = \mathbf{\frac{\pi}{4}}$

11. **Given.** $|\vec{a} + \vec{b}| = 60$ and $|\vec{a} - \vec{b}| = 40$

Using formula, $|\vec{a} + \vec{b}|^2 + |\vec{a} - \vec{b}|^2 = 2(|\vec{a}|^2 + |\vec{b}|^2)$

$\Rightarrow (60)^2 + (40)^2 = 2(22^2 + |\vec{b}|^2)$

$\Rightarrow 3600 + 1600 = 2(484 + |\vec{b}|^2)$

$\Rightarrow \frac{5200}{2} = 484 + |\vec{b}|^2$

$\Rightarrow 2600 - 484 = |\vec{b}|^2$

$\Rightarrow |\vec{b}|^2 = 2116 \quad \therefore |\vec{\boldsymbol{b}}| = \sqrt{2116} = \mathbf{46}$

12. Example of vectors $\vec{a}$ and $\vec{b}$ such that $|\vec{a}| = |\vec{b}|$ but $\vec{a} \neq \vec{b}$ are $\vec{\boldsymbol{a}} = \hat{\boldsymbol{i}}, \vec{\boldsymbol{b}} = \hat{\boldsymbol{j}}$.

13. Let γ be the required angle.

As we know, $l^2 + m^2 + n^2 = 1$

$\Rightarrow \left(\frac{1}{\sqrt{3}}\right)^2 + \left(\frac{1}{6}\right)^2 + n^2 = 1 \quad \Rightarrow \frac{1}{3} + \frac{1}{6} + n^2 = 1$

$\Rightarrow n^2 = 1 - \frac{1}{3} - \frac{1}{6} = \frac{6-2-1}{6} = \frac{3}{6} = \frac{1}{2} \quad \Rightarrow n = \frac{1}{\sqrt{2}}$

Now, $\cos\gamma = \frac{1}{\sqrt{2}} \quad \therefore \gamma = \mathbf{45°}$ or $\frac{\pi}{4}$

14. Direction ratios of the given line are 2, –1, 2.

Hence, Direction Cosines of the line are:

$$\frac{2}{\sqrt{(2)^2 + (-1)^2 + (2)^2}}, \frac{-1}{\sqrt{(2)^2 + (-1)^2 + (2)^2}}, \frac{2}{\sqrt{(2)^2 + (-1)^2 + (2)^2}}$$

or $\mathbf{\frac{2}{3}, \frac{-1}{3}, \frac{2}{3}}$.

15. We know that, $P(A' \cap B) = P(B) - P(A \cap B)$

$= P(B) - P(A)\,.\,P(B)$ ...**[∵ A and B are independent events**

$= [1 - P(A)]\,P(B) = P(A')\,P(B)$

Since, $P(A' \cap B) = P(A')P(B)$

Therefore A′ and B are independent events.

16. **P(Red transferred and red drawn or black transferred and red drawn)**

$= \frac{3}{8} \times \frac{7}{11} + \frac{5}{8} \times \frac{6}{11} = \frac{21}{88} + \frac{30}{88} = \mathbf{\frac{51}{88}}$

17. (*i*) (*c*); Let E_1, E_2 and E_3 denote the three types of flowers seeds.

Let E be the event that seed will germinate.

$$P(E_1) = \frac{4}{10} \text{ and } P(E|E_1) = \frac{45}{100}$$
$$P(E_2) = \frac{4}{10} \text{ and } P(E|E_2) = \frac{60}{100}$$
$$P(E_3) = \frac{2}{10} \text{ and } P(E|E_3) = \frac{35}{100}$$
$$P(E) = P(E_1)\,P(E|E_1) + P(E_2)\,P(E|E_2) + P(E_3)\,P(E|E_3)$$
$$= \frac{4}{10}\times\frac{45}{100} + \frac{4}{10}\times\frac{60}{100} + \frac{2}{10}\times\frac{35}{100}$$
$$= \frac{180}{1000} + \frac{240}{1000} + \frac{70}{1000} = \frac{490}{1000} = \frac{49}{1000} = \mathbf{0.49}$$

(ii) **(*b*)**; $P(\bar{E}|E_3) = 1 - P(E|E_3) = 1 - \frac{35}{100} = \frac{65}{100} = \mathbf{0.65}$

(iii) **(*c*)**; Here, $P(\bar{E}|E_1) = 1 - P(E|E_1) = 1 - \frac{45}{100} = \frac{55}{100}$

$$P(\bar{E}|E_2) = 1 - P(E|E_2) = 1 - \frac{60}{100} = \frac{40}{100}$$
$$P(E_2|\bar{E}) = \frac{P(E_2)P(\bar{E}|E_2)}{P(E_1)P(\bar{E}|E_1) + P(E_2)P(\bar{E}|E_2) + P(E_3)P(\bar{E}|E_3)}$$
$$= \frac{\frac{4}{10}\times\frac{40}{100}}{\frac{4}{10}\times\frac{55}{100}\times\frac{4}{10}\times\frac{40}{100}+\frac{2}{10}\times\frac{65}{100}} = \frac{16}{51}$$

(iv) **(*b*)**; $P(E_1|\bar{E}) = \frac{P(E_1)P(\bar{E}|E_1)}{P(E_1)P(\bar{E}|E_1) + P(E_2)P(\bar{E}|E_2) + P(E_3)P(\bar{E}|E_3)}$

$$= \frac{\frac{4}{10}\times\frac{55}{100}}{\frac{4}{10}\times\frac{55}{100}+\frac{4}{10}\times\frac{40}{100}+\frac{2}{10}\times\frac{65}{100}} = \frac{22}{51}$$

(v) **(*a*)**; $P(\bar{E}|E_1) = 1 - P(E|E_1) = \mathbf{\frac{55}{100}}$

18. ***(i)*** **(*b*)**; We have, A(1, 0) and B(2, 2)

Equation of the line I : $(y - y_1) = \frac{(y_2 - y_1)}{(x_2 - x_1)}(x - x_1)$

$\Rightarrow (y - 0) = \left(\frac{2-0}{2-1}\right)(x - 1)$ $\quad\therefore\ \mathbf{y = 2(x - 1)}$

(ii) **(*c*)**; We have, B(2, 2) and C(3, 1)

Equation of the line II : $(y - y_1) = \frac{(y_2 - y_1)}{(x_2 - x_1)}(x - x_1)$

$\Rightarrow (y - 2) = \left(\frac{1-2}{3-2}\right)(x - 2)$

$\Rightarrow (y - 2) = -1\,(x - 2)$ $\quad\Rightarrow y - 2 = -x + 2$ $\quad\therefore\ \mathbf{y = 4 - x}$

(iii) **(*d*)**; We have, A(1, 0) and C(3, 1)

Equation of the line III : $(y - y_1) = \frac{(y_2 - y_1)}{(x_2 - x_1)}(x - x_1)$

$\Rightarrow (y - 0) = \frac{1-0}{3-1}(x - 1)$ $\quad\Rightarrow 2y = x - 1$ $\quad\therefore\ \mathbf{y = \frac{x-1}{2}}$

(*iv*) (*a*); Area of $\Delta ABC = \int_1^2 2(x-1)\,dx + \int_2^3 (4-x)\,dx - \int_1^3 \frac{(x-1)}{2}$

$$= 2\left[\frac{x^2}{2} - x\right]_1^2 + \left[4x - \frac{x^2}{2}\right]_2^3 - \left[\frac{x^2}{4} - \frac{1}{2}x\right]_1^3$$

$$= 2\left[\frac{4}{2} - 2 - \frac{1}{2} + 1\right] + \left[12 - \frac{9}{2} - 8 + \frac{4}{2}\right] - \left[\frac{9}{4} - \frac{3}{2} - \frac{1}{4} + \frac{1}{2}\right]$$

$$= 2\left[\frac{1}{2}\right] + \left[4 - \frac{5}{2}\right] - [2-1]$$

$$= 1 + \frac{3}{2} - 1 = \mathbf{\frac{3}{2}\ sq.\ unit.}$$

(*v*) (*c*); The given lines are **intersecting**.

19. **L.H.S.** $\cos[\tan^{-1}\{\sin(\cot^{-1}x)\}] = \cos[\tan^{-1}\{\sin\theta\}]$ ...$\left[\text{Let } \cot^{-1}x = \theta \Rightarrow \cot\theta = \frac{x}{1} = \frac{B}{P}\right.$; $\therefore \sin\theta = \frac{P}{H} = \frac{1}{\sqrt{1+x^2}}$ **...[using Pythagoras theorem**

$$= \cos\left[\tan^{-1}\left(\frac{1}{\sqrt{1+x^2}}\right)\right]$$

$= \cos A = \sqrt{\frac{1+x^2}{2+x^2}}$ = **R.H.S. (Hence Proved)** ...$\left[\text{Let } \tan^{-1}\frac{1}{\sqrt{1+x^2}} = A \Rightarrow \tan A = \frac{1}{\sqrt{1+x^2}} = \frac{P}{B}\right.$; $\therefore \cos A = \frac{B}{H} = \frac{\sqrt{1+x^2}}{\sqrt{2+x^2}}$ **...[using Pythagoras theorem**

20. Let $A = \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix}$

Given. $PA = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$ $\Rightarrow P = A^{-1}\begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$ ...(*i*)

As we know, $A^{-1} = \frac{1}{|A|}$ adjoint A

Here, $|A| = \begin{vmatrix} -3 & 2 \\ 5 & -3 \end{vmatrix} = 9 - 10 = -1$

and Adjoint $A = \begin{bmatrix} -3 & -2 \\ -5 & -3 \end{bmatrix}$

$\therefore$ $\mathbf{A^{-1}} = \frac{1}{-1}\begin{bmatrix} -3 & -2 \\ -5 & -3 \end{bmatrix} = \begin{bmatrix} 3 & 2 \\ 5 & 3 \end{bmatrix}$ ...(*ii*)

Now, $P = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} 3 & 2 \\ 5 & 3 \end{bmatrix}$ **...[From (*i*) and (*ii*)**

$\therefore$ $\mathbf{P} = \begin{bmatrix} 3+10 & 2+6 \\ 6-5 & 4-3 \end{bmatrix} = \begin{bmatrix} \mathbf{13} & \mathbf{8} \\ \mathbf{1} & \mathbf{1} \end{bmatrix}$

Or

We have, $A^2 = 5A + kI$

$$\Rightarrow \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} = 5\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} + k\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 9-1 & 3+2 \\ -3-2 & -1+4 \end{bmatrix} = \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} + \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} = \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -7 & 0 \\ 0 & -7 \end{bmatrix} = \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix}$$

$\therefore \quad k = -7$

21. Let $x = \sin A \Rightarrow \sin^{-1}x = A$ ...(*i*)

and $y = \sin B \Rightarrow \sin^{-1}y = B$ ...(*ii*)

We have, $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$

$\Rightarrow \quad \sqrt{1-\sin^2 A} + \sqrt{1-\sin^2 B} = a(\sin A - \sin B)$ **...[From (*i*) & (*ii*)**

$\Rightarrow \quad \cos A + \cos B = a(\sin A - \sin B)$ **...[$\because -\sin^2\theta = \cos^2\theta$**

$$\Rightarrow \quad 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right) = a\left[2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)\right]$$

$$\Rightarrow \quad \cos\left(\frac{A-B}{2}\right) = a\sin\left(\frac{A-B}{2}\right) \qquad \Rightarrow \cot\left(\frac{A-B}{2}\right) = a$$

$$\Rightarrow \quad \left(\frac{A-B}{2}\right) = \cot^{-1}a \qquad \Rightarrow A - B = 2\cot^{-1}a$$

$\therefore \quad \sin^{-1}x - \sin^{-1}y = 2\cot^{-1}a$

Differentiating both sides w.r.t. x, we have

$$\frac{1}{\sqrt{1-x^2}} - \frac{1}{\sqrt{1-y^2}}\frac{dy}{dx} = 0 \qquad \Rightarrow \frac{1}{\sqrt{1-y^2}}\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$$

$\therefore \quad \boldsymbol{\frac{dy}{dx} = \frac{\sqrt{1-y^2}}{\sqrt{1-x^2}}}$ **(Hence Proved)**

22. **Given.** $f(x) = 3x^4 - 4x^3 - 12x^2 + 5$

Differentiating both sides w.r.t. x, we get

$f'(x) = 12x^3 - 12x^2 - 24x = 12x(x^2 - x - 2) = 12x(x^2 - 2x + x - 2)$

$= 12x(x(x-2) + 1(x-2)) = 12x(x+1)(x-2)$

When $f'(x) = 0$ $\quad \therefore \quad x = -1, 0, 2$

Intervals	*Checking points*	*Values of*				*Sign of $f'(x)$*	*Nature of $f(x)$*
		12	*x*	*$(x+1)$*	*$(x-2)$*		
$(-\infty, -1)$	-2	+	–	–	–	<0	Strictly decreasing
$(-1, 0)$	-0.5	+	–	+	–	> 0	Strictly increasing
$(0, 2)$	1	+	+	+	–	< 0	Strictly decreasing
$(2, \infty)$	3	+	+	+	+	> 0	Strictly increasing

Therefore, $f(x)$ is strictly increasing in $(-1, 0)$ and $(2, \infty)$

$f(x)$ is strictly decreasing in $(-\infty, -1)$ and $(0, 2)$.

23. $\int \frac{1+\sin 2x}{1+\cos 2x} \cdot e^{2x}\, dx = \frac{1}{2}\int \frac{1+\sin p}{1+\cos p} \cdot e^{p}\, dp$ ...$\left[\text{Let } p = 2x \Rightarrow \frac{dp}{2} = dx\right.$

$$= \frac{1}{2}\int \frac{1+2\sin\frac{p}{2}\cos\frac{p}{2}}{2\cos^2\frac{p}{2}} \cdot e^{p}\, dp = \frac{1}{2}\int\left(\frac{1}{2\cos^2\frac{p}{2}} + \frac{2\sin\frac{p}{2}\cos\frac{p}{2}}{2\cos^2\frac{p}{2}}\right) e^{p}\, dp$$ ...$\left[\because \sin x = 2\sin x \cos x,\ 1+\cos x = 2\cos^2\frac{x}{2}\right.$

$$= \frac{1}{2}\int\left(\frac{1}{2}\sec^2\frac{p}{2} + \tan\frac{p}{2}\right)e^{p}\, dp = \frac{1}{2}\int [f'(p) + f(p)]\, e^{p}\, dp$$ ...$\left[\text{where } f(p) = \tan\frac{p}{2},\ f'(p) = \frac{1}{2}\sec^2\frac{p}{2}\right.$

$$= \frac{1}{2} f(p) \cdot e^{p} + C = \frac{1}{2}\tan\frac{p}{2} \cdot e^{p} + C = \frac{1}{2}\tan\left(\frac{2x}{2}\right) \cdot e^{2x} + C$$

$$= \frac{1}{2} e^{2x} \cdot \tan x + C$$

Or

$$\int \frac{\sin\phi}{\sqrt{\sin^2\phi + 2\cos\phi + 3}}\, d\phi = \int \frac{\sin\phi}{\sqrt{1-\cos^2\phi + 2\cos\phi + 3}}\, d\phi = \int \frac{\sin\phi}{4-\cos^2\phi + 2\cos\phi}\, d\phi$$

$$= -\int \frac{dp}{\sqrt{4 - p^2 + 2p}}$$ ...$\left[\text{Let } p = \cos\phi,\ dp = -\sin\phi\, d\phi,\ -dp = \sin\phi\, d\phi\right.$

$$= -\int \frac{dp}{\sqrt{-(p^2 - 2p - 4)}} = -\int \frac{dp}{\sqrt{-(p^2 - 2p + 1^2 - 4 - 1)}}$$

$$= -\int \frac{dp}{\sqrt{-[(p-1)^2 - 5]}} = -\int \frac{dp}{\sqrt{\sqrt{5}^2 - (p-1)^2}}$$

$$= -\sin^{-1}\left(\frac{p-1}{\sqrt{5}}\right) + c$$ ...$\left[\because \int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\frac{x}{a} + c\right.$

$$= -\sin^{-1}\left(\frac{\cos\phi - 1}{\sqrt{5}}\right) + c$$

24. Given equations of curve and line are

Parabola, $x^2 = 4y \Rightarrow y = \frac{x^2}{4}$ ...(*i*)

x	0	±2
y	0	1

and line, $x = 4y - 2 \Rightarrow y = \frac{x+2}{4}$ ...(*ii*)

x	0	−2
y	0·5	0

For point of intersection, we have $x = 4y - 2 \Rightarrow x = x^2 - 2$ ...[From (*i*)

$\Rightarrow x^2 - x - 2 = 0 \Rightarrow x^2 - 2x + x - 2 = 0$

$\Rightarrow x(x-2) + 1(x-2) = 0$

$\Rightarrow (x-2)(x+1) = 0 \quad \therefore x = 2, \text{ or } x = -1$

When $x = 2$, $y = 1$ ...[using (*i*)

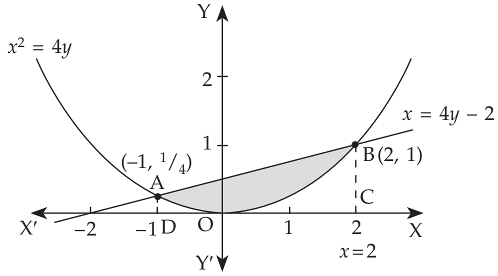
When $x = -1$, $y = \frac{1}{4}$ ...[using (*ii*)

The points of intersection are A $\left(-1, \frac{1}{4}\right)$ and B (2, 1).

Area of the shaded region = Area of the region (ABCODA – ABOA)

$$= \int_{-1}^{2} [y\,(\text{line}) - y\,(\text{parabola})]\, dx = \int_{-1}^{2}\left[\frac{x+2}{4} - \frac{x^2}{4}\right] dx$$

$$= \frac{1}{4}\int_{-1}^{2} (x + 2 - x^2)\, dx = \frac{1}{4}\left[\frac{x^2}{2} + 2x - \frac{x^3}{3}\right]_{-1}^{2}$$



$$= \frac{1}{4}\left[\left(\frac{4}{2} + 4 - \frac{8}{3}\right) - \left(\frac{1}{2} - 2 + \frac{1}{3}\right)\right]$$

$$= \frac{1}{4}\left[\left(2 + 4 - \frac{8}{3}\right) - \left(\frac{3 - 12 + 2}{6}\right)\right]$$

$$= \frac{1}{4}\left[6 - \frac{8}{3} + \frac{7}{6}\right] = \frac{1}{4}\left[\frac{36 - 16 + 7}{6}\right]$$

$$= \frac{1}{4} \times \frac{27}{6} = \frac{9}{8} \textbf{ sq. units}$$

25. The given lines are

$$\vec{r_1} = 6\hat{i} + 2\hat{j} + 2\hat{k} + \lambda(\hat{i} - 2\hat{j} + 2\hat{k}) \text{ and } \vec{r_2} = -4\hat{i} - \hat{k} + \mu(3\hat{i} - 2\hat{j} - 2\hat{k})$$

On comparing the above with $\vec{r} = \vec{a} + \lambda\vec{b}$

Here $a_1 = 6\hat{i} + 2\hat{j} + 2\hat{k}$, $\vec{b_1} = \hat{i} - 2\hat{j} + 2\hat{k}$

$a_2 = -4\hat{i} - \hat{k}$, $\vec{b_2} = 3\hat{i} - 2\hat{j} - 2\hat{k}$

$$\vec{b_1} \times \vec{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 2 \\ 3 & -2 & -2 \end{vmatrix} = \hat{i}(4 + 4) - \hat{j}(-2 - 6) + \hat{k}(-2 + 6) = 8\hat{i} + 8\hat{j} + 4\hat{k}$$

$$|\vec{b_1} \times \vec{b_2}| = \sqrt{(8)^2 + (8)^2 + (4)^2} = \sqrt{64 + 64 + 16} = \sqrt{144} = 12$$

$$\vec{a_2} - \vec{a_1} = -4\hat{i} - \hat{k} - 6\hat{i} - 2\hat{j} - 2\hat{k}$$
$$= -10\hat{i} - 2\hat{j} - 3\hat{k} = -(10\hat{i} + 2\hat{j} + 3\hat{k})$$

$\therefore$ **Shortest Distance** $= \left|\frac{(\vec{b_1} \times \vec{b_2}).(\vec{a_2} - \vec{a_1})}{|\vec{b_1} \times \vec{b_2}|}\right|$

$$= \left|\frac{-(8\hat{i} + 8\hat{j} + 4\hat{k}).(10\hat{i} + 2\hat{j} + 3\hat{k})}{12}\right|$$

$$= \frac{80 + 16 + 12}{12} = \frac{108}{12} = 9 \textbf{ units.}$$

26. Let $\vec{a}, \vec{b}, \vec{c}$ be three unit vectors such that $\vec{a} + \vec{b} = \vec{c}$.

Let $\left.\begin{array}{l} |\vec{a}| = |\vec{b}| = 1 \\ |\vec{a} + \vec{b}| = 1 \end{array}\right]$ ...[Given

$|\vec{a} + \vec{b}|^2 = (\vec{a} + \vec{b})^2$ $\left[\because |\vec{x}|^2 = \vec{x}^2\right.$

$(1)^2 = (\vec{a} + \vec{b}).(\vec{a} + \vec{b})$

$$1 = \vec{a}.\vec{a} + \vec{a}.\vec{b} + \vec{b}.\vec{a} + \vec{b}.\vec{b}$$

$$1 = |\vec{a}|^2 + 2\vec{a}.\vec{b} + |\vec{b}|^2$$

$\left[\because \vec{a}.\vec{a} = \vec{a}^2 = \vec{a}^2,\ \vec{a}.\vec{b} = \vec{b}.\vec{a}\right.$

$$1 = (1)^2 + 2\vec{a}.\vec{b} + (1)^2$$

$$1 - 1 - 1 = 2\vec{a}.\vec{b}$$

$$-1 = 2\vec{a}.\vec{b} \qquad ...(i)$$

Now $|\vec{a} - \vec{b}|^2 = (\vec{a} - \vec{b})^2 = (\vec{a} - \vec{b}).(\vec{a} - \vec{b})$

$= \vec{a}.\vec{a} - \vec{a}.\vec{b} - \vec{b}.\vec{a} + \vec{b}.\vec{b}$

$= |\vec{a}|^2 - 2\vec{a}.\vec{b} + |\vec{b}|^2$

$= (1)^2 - (-1) + (1)^2 = 3$ **[from (*i*)]**

$\therefore \quad |\vec{a} - \vec{b}| = \sqrt{3}$

27. Let the given equations of lines are

$$\frac{x+2}{1} = \frac{y-3}{2} = \frac{z+1}{4} = \lambda \text{ and } \frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4} = \mu$$

So, the General points of above lines are $(\lambda - 2, 2\lambda + 3, 4\lambda - 1)$ and $(2\mu + 1, 3\mu + 2, 4\mu + 3)$
The required line passes through the point (1, 1, 1).

$\therefore$ Direction Ratios of required lines are:
$(\lambda - 2 - 1, 2\lambda + 3 - 1, 4\lambda - 1 - 1)$ and $(2\mu + 1 - 1, 3\mu + 2 - 1, 4\mu + 3 - 1)$

i.e., $(\lambda - 3, 2\lambda + 2, 4\lambda - 2)$ and $(2\mu, 3\mu + 1, 4\mu + 2)$

$$\therefore \quad \frac{\lambda - 3}{2\mu} = \frac{2\lambda + 2}{3\mu + 1} = \frac{4\lambda - 2}{4\lambda + 2} = K \text{ (say)}$$

$\Rightarrow \quad \lambda - 3 = 2\mu K,\ 2\lambda + 2 = (3\mu + 1)K,\ 2\lambda - 1 = (2\mu + 1)K$

$\Rightarrow \quad \dfrac{\lambda - 3}{2} = \mu K \qquad ...(i)$

$2\lambda + 2 = 3\mu K + K,$

$2\lambda + 2 = 3\left(\dfrac{\lambda - 3}{2}\right) + K$ **...[From (*i*)**

$4\lambda + 4 = 3\lambda - 9 + 2K$

$\lambda - 2K = -13 = 0 \qquad ...(ii)$

$2\lambda - 1 = (2\mu + 1)K$

$2\lambda - 1 = 2\mu K + K$

$2\lambda - 1 = 2\left(\dfrac{\lambda - 3}{2}\right) + K$ **...[From (*i*)**

$2\lambda - 1 = \lambda - 3 + K$

$\lambda - K = -2 \qquad ...(iii)$

Solving (*ii*) & (*iii*), we get $\mathbf{K = 11}$ **and** $\boldsymbol{\lambda = 9}$

From (*i*), $\mu K = \dfrac{\lambda - 3}{2} \quad \Rightarrow 11\mu = \dfrac{9 - 3}{2}$

$\Rightarrow \quad 22\mu = 6 \qquad \Rightarrow \mu = \dfrac{3}{11}$

$\therefore$ The Direction Ratios of the required lines are: **(6, 20, 34) or (3, 10, 17).**

Hence, the required equations of the line passing through the point is $\dfrac{x-1}{3} = \dfrac{y-1}{10} = \dfrac{z-1}{17}$.

28.

Number of Good Oranges	=	16
Number of Bad Oranges	=	04
Total Oranges	**=**	**20**

Let X denote the random variable, so X can take the values 0, 1, 2.

$$P(X=0) = \frac{^{16}C_2}{^{20}C_2} = \frac{60}{95}$$

$$P(X=1) = \frac{^{4}C_1 \times {^{16}C_1}}{^{20}C_2} = \frac{32}{95}$$

$$P(X=2) = \frac{^{4}C_2}{^{20}C_2} = \frac{3}{95}$$

Probability distribution is

X_i	0	1	2	Total
P_i	60/95	32/95	3/95	1
X_iP_i	0	32/95	6/95	38/95
$X_i^2P_i$	0	32/95	12/95	44/95

Or

Let E_1 be the event that six occurs.
Let E_2 be the event that six does not occur.
Let A be the event that a man throws a die and reports that it is a six.

$\therefore \quad P(E_1) = P(\text{a no. } 6) = \frac{1}{6}, \qquad P(A|E_1) = \frac{3}{4}$

$\therefore \quad P(E_2) = 1 - \frac{1}{6} = \frac{5}{6}, \qquad P(A|E_2) = \frac{1}{4}$

Using Baye's theorem,

Required probability, $P(E_1|A)$ $= \frac{P(E_1)\times P(A|E_1)]}{[P(E_1)\times P(A|E_1)] + P(E_2)\times P(A|E_2)]}$

$$= \frac{\frac{1}{6}\times\frac{3}{4}}{\left(\frac{1}{6}\times\frac{3}{4}\right)+\left(\frac{5}{6}\times\frac{1}{4}\right)} = \frac{\frac{3}{24}}{\frac{3}{24}+\frac{5}{24}} = \frac{\frac{3}{24}}{\frac{3+5}{24}} = \mathbf{\frac{3}{8}}$$

29. **Reflexive:** R is reflexive, as $1 + a.a = 1 + a^2 > 0$

$\Rightarrow \quad (a, a) \in R \ \forall \ a \in R$

Symmetric: If $(a, b) \in R$

then, $1 + ab > 0$

$\Rightarrow \quad 1 + ba > 0$

$\Rightarrow \quad (b, a) \in R$

Hence, r is symmetric.

Transitive:

Let $a = -8, b = -1, c = \frac{1}{2}$

Since, $1 + ab = 1 + (-8)(-1) = 9 > 0 \qquad \therefore (a, b) \in R$

also, $1 + bc = 1 + (-1)\left(\frac{1}{2}\right) = \frac{1}{2} > 0 \qquad \therefore (b, c) \in R$

But, $1 + ac = 1 + (-8)\left(\frac{1}{2}\right) = -3 < 0$

Hence, R is not transitive.

30. Since, f is differentiable at $x = 2$, therefore, f is continous at $x = 2$

$$\text{L.H.L.} = \lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} x^2 = (2)^2 = 4$$

$$\text{R.H.L} = \lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} (ax + b) = 2a + b$$

At $x = 2, f(x) = x^2$

$\boldsymbol{f(2)} = 2^2 = 4$

Since, f is continuous at $x = 2$,

$\therefore$ **L.H.L. = R.H.L. =** $\boldsymbol{f(2)}$

$4 = 2a + b = 4 \quad \Rightarrow \quad 2a + b = 4 \quad \therefore \quad b = 4 - 2a$ ...(*i*)

Since, f is differentiable at $x = 2$,

$$Lf'(2) = \lim_{x\to 2^-} \frac{f(x) - f(2)}{x - 2} = \lim_{x\to 2^-} \frac{x^2 - 4}{(x - 2)} = \lim_{x\to 2^-} \frac{(x - 2)(x + 2)}{(x - 2)} = 2 + 2 = 4$$

$$Rf'(2) = \lim_{x\to 2^+} \frac{f(x) - f(2)}{x - 2} = \lim_{x\to 2^+} \frac{(ax + b) - 4}{x - 2} = \lim_{x\to 2^+} \frac{ax + 4 - 2a - 4}{x - 2} \text{ [From (i)]} = \lim_{x\to 2^+} \frac{a(x - 2)}{(x - 2)} = a$$

Since, f is differentiable at $x = 2$, $Lf'(2) = Rf'(2) \Rightarrow 4 = a$

From (*i*), $b = 4 - 2(4) = 4 - 8 = -4$ $\quad \therefore$ $\boldsymbol{a = 4, b = -4}$

31. Let $y = (\log x)^x + x^{x \cos x}$

Let $y = A + B$

Differentiating both sides w.r.t. x

$$\frac{dy}{dx} = \frac{dA}{dx} + \frac{dB}{dx} \quad \text{...(i)}$$

Here, $A = (\log x)^x$

Taking log on both the sides

$\log A = \log (\log x)^x$

$\log A = x \, . \log (\log x)$

Differentiating both sides w.r.t. x

$$\frac{1}{A} \cdot \frac{dA}{dx} = x\left[\frac{1}{\log x} \cdot \frac{1}{x}\right] + \log(\log x) \, . \, 1$$

$$\frac{dA}{dx} = A\left[\frac{1}{\log x} + \log(\log x)\right]$$

$$\frac{dA}{dx} = (\log x)^x \left[\frac{1}{\log x} + \log(\log x)\right] \quad \text{...(ii)}$$

Now, $B = x^{x \cos x}$

Taking log on both sides

$\log B = \log x^{x \cos x}$

$\log B = x \cos x \, . \log x$

Differentiating both sides w.r.t. x

$$\frac{1}{B} \cdot \frac{dB}{dx} = x \cos x \, . \, \frac{1}{x} + x \log x \, . \, (-\sin x) + \cos x \, . \log x \, . \, 1$$

$$\frac{dB}{dx} = B\,[\cos x - x \log x \,.\, \sin x + \cos x \,.\, \log x]$$

$$\frac{dB}{dx} = x^{x\cos x}\,[\cos x - x \log x \,.\, \sin x + \cos x \,.\, \log x] \quad \ldots(iii)$$

From (*i*), (*ii*) and (*iii*), we have

$$\frac{dy}{dx} = (\log x)^x \left[\frac{1}{\log x} + \log(\log x)\right] + x^{x\cos x}\,[\cos x - x \log x \,.\, \sin x + \cos x \,.\, \log x]$$

Or

We have, $x = a \sin pt$ and $y = b \cos pt$

Differentiating w.r.t. t, we have

$$\frac{dx}{dt} = a \,.\, \cos pt \,.\, p \qquad \frac{dy}{dt} = -b \sin pt \,.\, p$$

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx} = \frac{-bp \sin pt}{ap \cos pt}$$

$$\therefore \quad \frac{dy}{dx} = \frac{-b}{a} \tan pt$$

Differentiating w.r.t. x, we have

$$\frac{d^2y}{dx^2} = \frac{-b}{a} \,.\, \sec^2 pt \,.\, p \,.\, \frac{dt}{dx}$$

$$= \frac{-bp}{a} \sec^2 pt \times \frac{1}{ap \cos pt} = \frac{-b}{a^2} \sec^3 pt$$

$$\therefore \quad \left.\frac{d^2y}{dx^2}\right]_{\text{at } t=0} = \frac{-b}{a^2} \sec^3(0) = \frac{-b}{a^2}(1) = \frac{-b}{a^2}$$

32. Let the normal be at (x_1, y_1) to the curve $2y = x^2$ $\ldots(i)$

$$2y = x^2 \qquad \Rightarrow y = \frac{1}{2}x^2$$

Differentiating both sides w.r.t. x, we have $\frac{dy}{dx} = \frac{1}{2}.2x = x$

Slope of normal at $(x_1, y_1) = \dfrac{-1}{\left(\frac{dy}{dx}\right)_{(x_1, y_1)}} = \dfrac{-1}{x_1}$

Equation of normal at (x_1, y_1) is

$$y - y_1 = \text{slope of normal } (x - x_1)$$

$$y - y_1 = \frac{-1}{x_1}(x - x_1) \quad \ldots(ii)$$

Point (2, 1) lies on (*ii*), $1 - y_1 = \dfrac{-1}{x_1}(2 - x_1)$

$$\Rightarrow \quad x_1 - x_1y_1 = -2 + x_1 \Rightarrow x_1y_1 = 2 \quad \ldots(iii)$$

Also, point (x_1, y_1) lies on the given curve (*i*), we have

$$2y_1 = x_1^2 \qquad \Rightarrow y_1 = \frac{1}{2}x_1^2$$

Putting the value of y_1 in (*iii*), we have

$$x_1\left(\frac{1}{2}x_1^2\right) = 2 \quad \Rightarrow x_1^3 = 2^2$$

Taking cube root on both sides, $x_1 = 2^{2/3}$

From (*iii*), $2^{2/3}.y_1 = 2 \quad \Rightarrow y_1 = \frac{2}{2^{2/3}} = 2^{1/3}$

Putting the value of x_1 and y_1 in (*i*), we have

$$y - 2^{1/3} = \frac{-1}{2^{2/3}}(x - 2^{2/3})$$

$$2^{2/3}y - 2 = -x + 2^{2/3}$$

Therefore, $x + 2^{2/3}y = 2 + 2^{2/3}$ is the required equation of the normal.

33. Let $I = \int_0^{\frac{\pi}{2}} \frac{x\sin x\cos x}{\sin^4 x + \cos^4 x}dx$ ...(*i*)

$$= \int_0^{\pi/2} \frac{\left(\frac{\pi}{2}-x\right)\sin\left(\frac{\pi}{2}-x\right).\cos\left(\frac{\pi}{2}-x\right)}{\sin^4\left(\frac{\pi}{2}-x\right)+\cos^4\left(\frac{\pi}{2}-x\right)}dx \qquad \left[\because \int_0^a f(x)dx = \int_0^a f(a-x)dx\right.$$

$$I = \int_0^{\pi/2} \frac{\left(\frac{\pi}{2}-x\right)\cos x\sin x}{\sin^4 x + \cos^4 x}dx \qquad ...(ii)$$

Adding (*i*) and (*ii*), we get

$$2I = \int_0^{\pi/2} \frac{x\sin x\cos x + \left(\frac{\pi}{2}-x\right)\cos x\sin x}{\sin^4 x + \cos^4 x}dx$$

$$2I = \int_0^{\pi/2} \frac{\frac{\pi}{2}\sin x\cos x}{\sin^4 x + \cos^4 x}dx$$

Dividing numerator and denominator by $\cos^4 x$, we get

$$2I = \frac{\pi}{2}\int_0^{\pi/2} \frac{\tan x.\sec^2 x}{\tan^4 x + 1}dx$$

$$\left[\begin{array}{l}\text{Let } p = \tan^2 x \\ dp = 2\tan x\sec^2 x\,dx \\ \frac{dp}{2} = \tan x\sec^2 x\,dx \\ \text{when } x = \frac{\pi}{2}, p = \infty \\ \text{when } x = 0, p = 0\end{array}\right.$$

$$= \frac{\pi}{2}\times\frac{1}{2}\int_0^{\infty}\frac{dp}{p^2+1}$$

$$= \frac{\pi}{4}\left[\tan^{-1}(p)\right]_0^{\infty}$$

$$2I = \frac{\pi}{4}[\tan^{-1}(\infty) - \tan^{-1}(0)]$$

$$\therefore \quad I = \frac{\pi}{8}\left[\frac{\pi}{2} - 0\right] = \frac{\pi^2}{16}$$

34. We have, $x + 2y = 2$

$\Rightarrow \quad x = 2 - 2y$

x	0	2
y	1	0

$y - x = 1$

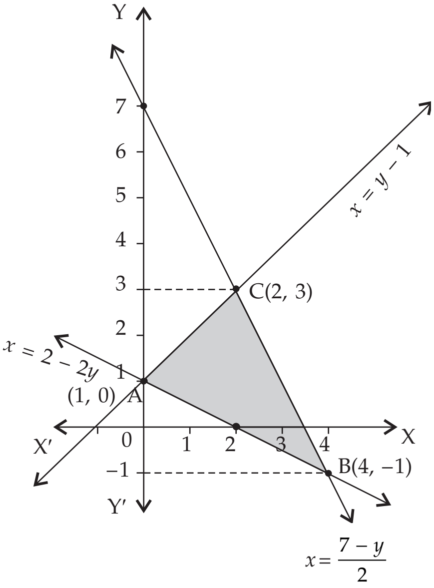
$\Rightarrow \quad y - 1 = x$

x	0	2
y	1	3

$2x + y = 7$

$\Rightarrow \quad x = \frac{7-y}{2}$

x	2	4
y	3	-1



$\therefore$ Points of intersection are A(1, 0), B(4, −1) and C(2, 3).

Area of shaded region $= \int_{-1}^{3} \frac{7-y}{2}\,dy - \int_{-1}^{1} (2-2y)\,dy - \int_{1}^{3} (y-1)\,dy$

$$= \frac{1}{2}\left[7y - \frac{y^2}{2}\right]_{-1}^{3} - \left[2y - \frac{2y^2}{2}\right]_{-1}^{1} - \left[\frac{y^2}{2} - y\right]_{1}^{3}$$

$$= \frac{1}{2}\left[\left(7(3) - \frac{9}{2}\right) - \left(-7 - \frac{1}{2}\right)\right] - [(2-1) - (-2-1)] - \left[\left(\frac{9}{2} - 3\right) - \left(\frac{1}{2} - 1\right)\right]$$

$= 12 - 4 - 2 =$ **6 sq. units.**

Or

The given lines are

$2x + y = 4,$

$\Rightarrow \quad y = 4 - 2x$

x	0	2
y	4	0

$3x - 2y = 6$

$\Rightarrow \quad 3x - 6 = 2y$

$\Rightarrow \quad y = \frac{3x-6}{2}$

x	0	2
y	-3	0

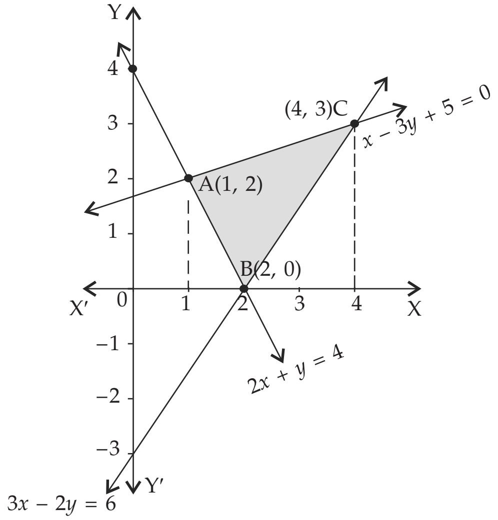
$x - 3y + 5 = 0$

$\Rightarrow \quad x + 5 = 3y$

$\Rightarrow \quad y = \frac{x+5}{3}$

x	1	4
y	2	3

The given lines intersect at the following points to form a triangle A(1, 2), B(2, 0), C(4, 3).



Area of shaded region = Area under segment (AC – AB – BC)

$$= \int_1^4 \left(\frac{x+5}{3}\right)dx - \int_1^2 (4-2x)\,dx - \int_2^4 \left(\frac{3x-6}{2}\right)dx$$

$$= \frac{1}{3}\left[\frac{x^2}{2} + 5x\right]_1^4 - \left[4x - \frac{2x^2}{2}\right]_1^2 - \frac{1}{2}\left[\frac{3x^2}{2} - 6x\right]_2^4$$

$$= \frac{1}{3}\left[\frac{4^2}{2} + 5(4) - \left(\frac{1}{2} + 5(1)\right)\right] - [4(2) - 2^2 - (4(1) - 1^2)] - \frac{1}{2}\left[\left(\frac{3}{2}(4)^2 - 6(4)\right) - \left(\frac{3}{2}(2)^2 - 6(2)\right)\right]$$

$$= \frac{1}{3}\left[8 + 20 - \frac{1}{2} - 5\right] - [8 - 4 - 4 + 1] - \frac{1}{2}\left[\frac{3}{2}(16) - 24 - \frac{3}{2}(4) + 12\right]$$

$$= \frac{1}{3}\left(23 - \frac{1}{2}\right) - (1) - \frac{1}{2}(24 - 24 - 6 + 12) = \frac{1}{3}\left(\frac{45}{2}\right) - (1) - \frac{1}{2}(6)$$

$$= \frac{15}{2} - 1 - 3 = \frac{15}{2} - 4 = \frac{15-8}{2} = \frac{7}{2}$$ **sq. units**

35. $(1 + y + x^2y)\,dx + (x + x^3)\,dy = 0$

$x(1 + x^2)dy = -(1 + y + x^2y)\,dx$

$$\frac{dy}{dx} = \frac{-[1 + y(1 + x^2)]}{x(1 + x^2)}$$

$$\frac{dy}{dx} = \frac{-1}{x(1 + x^2)} - \frac{y(1 + x^2)}{x(1 + x^2)}$$

$$\frac{dy}{dx} + \frac{y}{x} = \frac{-1}{x(1 + x^2)}$$

This is a linear differential equation of the form $\frac{dy}{dx} + Py = Q$.

Here, 'P' $= \frac{1}{x}$, Q $= \frac{-1}{x(1 + x^2)}$

$\therefore$ I.F. $= e^{\int P\,dx} = e^{\int \frac{1}{x}dx} = e^{\log x} = x$

Hence the solution is

$$y(\text{I.F.}) = \int Q.(\text{I.F.})\,dx$$

$$y(x) = \int \frac{-1}{x(1+x^2)}.x\,dx$$

$$xy = -\frac{dx}{1+x^2}$$

$$xy = -\tan^{-1}(x) + c \qquad \ldots(i)$$

$$1(0) = -\tan^{-1}(1) + c \qquad [\because y = 0; x = 1 \text{ (given)}]$$

$$0 = -\frac{\pi}{4} + c \qquad \Rightarrow \frac{\pi}{4} = c$$

Putting the value of c in (i), we get

$$xy = -\tan^{-1} x + \frac{\pi}{4} \quad \text{or} \quad \boldsymbol{xy + \tan^{-1} x = \frac{\pi}{4}}$$

36. Let $A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}$, $B = \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$

$$AB = \begin{bmatrix} -2-9+12 & 0-2+2 & 1+3-4 \\ 0+18-18 & 0+4-3 & 0-6+6 \\ -6-18+24 & 0-4+4 & 3+6-8 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$$

$\Rightarrow$ $AB = I$

$\Rightarrow$ $A^{-1}AB = A^{-1}I$ ...**[Pre multiplying by A^{-1}]**

$\Rightarrow$ $IB = A^{-1}$ $\quad \therefore \; B = A^{-1}$ ...(i)

Writing the given equations in matrix form

$$\begin{bmatrix} 1 & 0 & 3 \\ -1 & 2 & -2 \\ 2 & -3 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ 4 \\ -3 \end{bmatrix}$$

$$A^t X = C$$

$$(A^t)^{-1} \times A^t X = (A^t)^{-1} C \qquad [\because (A^t)^{-1} = (A^{-1})^t = B^t$$

$$X = (A^t)^{-1} C \qquad \textbf{[From (}i\textbf{)}$$

$$X = B^t C \qquad \textbf{[From (}i\textbf{)}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -2 & 9 & 6 \\ 0 & 2 & 1 \\ 1 & -3 & -2 \end{bmatrix}\begin{bmatrix} 9 \\ 4 \\ -3 \end{bmatrix} = \begin{bmatrix} -18+36-18 \\ 0+8-3 \\ 9-12+6 \end{bmatrix} = \begin{bmatrix} 0 \\ 5 \\ 3 \end{bmatrix}$$

$\therefore$ $\boldsymbol{x = 0, y = 5, z = 3}$

Or

Part I : Let $AB = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}\begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$

$$= \begin{bmatrix} -2-9+12 & 0-2+2 & 1+3-4 \\ 0+18-18 & 0+4-3 & 0-6+6 \\ -6-18+24 & 0-4+4 & 3+6-8 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$AB = I$$
$$A^{-1}(AB) = A^{-1} I$$
$$IB = A^{-1}$$
$$B = A^{-1} \quad \ldots(i)$$

Part II: Given equations can be written in matrix form as

$$AX = C$$
$$X = A^{-1} C$$
$$X = BC \quad \ldots\text{[from } (i)\text{]}$$

where $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}, C = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$

$$\therefore \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} -2+0+2 \\ 9+2-6 \\ 6+1-4 \end{bmatrix} = \begin{bmatrix} 0 \\ 5 \\ 3 \end{bmatrix}$$

$\therefore$ $\boldsymbol{x = 0, y = 5}$ **and** $\boldsymbol{z = 3}$

37. (1) Let a, b, c be the direction ratios (D.rs) of the normal to the required plane

Equation of plane through the point (1, 1, 1) is

$$a(x-1) + b(y-1) + c(z-1) = 0 \quad \ldots(i)$$

Since the plane (i) contains the line

$$\vec{r} = (-3\hat{i} + \hat{j} + 5\hat{k}) + \lambda(3\hat{i} - \hat{j} - 5\hat{k}),$$

$\therefore$ Point (–3, 1, 5) on the line also lies on the plane (i)

$\therefore$ $a(-3-1) + b(1-1) + c(5-1) = 0$

$\Rightarrow$ $-4a + 0b + 4c = 0 \quad \ldots(ii)$

Also the given first line is perpendicular to the normal to the required plane.

Using $a_1a_2 + b_1b_2 + c_1c_2 = 0$

$\therefore$ $3a - b - 5c = 0 \quad \ldots(iii)$

Solving (ii) and (iii),

$$\frac{a}{0+4} = \frac{-b}{20-12} = \frac{c}{4-0}$$

$$\frac{a}{4} = \frac{b}{-8} = \frac{c}{4}$$

or $\quad \frac{a}{1} = \frac{b}{-2} = \frac{c}{1} = \lambda$ (let)

$\therefore$ $a = \lambda, b = -2\lambda, c = \lambda$

Putting the values of a, b, c in (i),

$$\lambda(x-1) - 2\lambda(y-1) + \lambda(z-1) = 0 \quad [\div \text{ both sides by } \lambda$$
$$(x-1) - 2(y-1) + (z-1) = 0$$
$$x - 1 - 2y + 2 + z - 1 = 0$$
$$\boldsymbol{x - 2y + z = 0} \quad \ldots(iv)$$

(2) Now we have to show that the plane (iv) contains the given 2nd line

$$\vec{r} = (-\hat{i} + 2\hat{j} + 5\hat{k}) + \lambda(\hat{i} - 2\hat{j} - 5\hat{k})$$

Point (–1, 2, 5) of 2nd line lies on the plane (iv)

$\therefore$ $x - 2y + z = 0 \quad \ldots(v)$

$-1 - 2(2) + 5 = 0$

$-5 + 5 = 0$, which is true.

D.rs of the 2nd line are 1, –2, –5

D.rs of normal to the plane (*iv*) are 1, –2, 1.

Now, $a_1a_2 + b_1b_2 + c_1c_2$

$1(1) - 2(-2) - 5(1) = 1 + 4 - 5 = 0$

$\therefore$ Given 2nd line is parallel to the plane. ...(*vi*)

From (*v*) and (*vi*), **the plane (*iv*) *i.e.* $x - 2y + z = 0$ contains the given 2nd line.**

Or

Here $\vec{a_1} = (-3\hat{i} + \hat{j} + 5\hat{k})$, $\vec{b_1} = -3\hat{i} + \hat{j} + 5\hat{k}$

$\vec{a_2} = -\hat{i} + 2\hat{j} + 5\hat{k}$, $\vec{b_2} = -\hat{i} + 2\hat{j} + 5\hat{k}$

Now, $\vec{a_2} - \vec{a_1} = -\hat{i} + 2\hat{j} + 5\hat{k} - (-3\hat{i} + \hat{j} + 5\hat{k}) = 2\hat{i} + \hat{j}$

$$(\vec{a_2} - \vec{a_1}) \times (\vec{b_1} \times \vec{b_2}) = \begin{vmatrix} 2 & 1 & 0 \\ -3 & 1 & 5 \\ -1 & 2 & 5 \end{vmatrix} = 2(5-10) - 1(-15+5) \quad \text{...[expanding along } R_1$$

$$= -10 + 10 = 0$$

Therefore, lines are co-planar.

$$\text{Perpendicular vector } (\vec{n}) \text{ to the plane} = \vec{b_1} \times \vec{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -3 & 1 & 5 \\ -1 & 2 & 5 \end{vmatrix} \quad \text{...[expanding along } R_1$$

$$= \hat{i}(5-10) - \hat{j}(-15+5) + \hat{k}(-6+1) = -5\hat{i} + 10\hat{j} - 5\hat{k}$$

$$= -5\hat{i} + 10\hat{j} - 5\hat{k}$$

Equation of plane is $\vec{r}.(\vec{b_1} \times \vec{b_2}) = \vec{a_1}.(\vec{b_1} \times \vec{b_2})$

$$\vec{r}.(-5\hat{i} + 10\hat{j} - 5\hat{k}) = (-3\hat{i} + \hat{j} + 5\hat{k}).(-5\hat{i} + 10\hat{j} - 5\hat{k})$$

$$\vec{r}.(-5\hat{i} + 10\hat{j} - 5\hat{k}) = 15 + 10 - 25$$

$$\vec{r}.(-5\hat{i} + 10\hat{j} - 5\hat{k}) = 0$$

or $\vec{r}.(\hat{i} - 2\hat{j} + \hat{k}) = 0$

$$(x\hat{i} + y\hat{j} + 2\hat{k}).(\hat{i} - 2\hat{j} + \hat{k}) = 0$$

$\therefore$ $\boldsymbol{x - 2y + z = 0}$

38. Maximize profit, P = ₹ $(8000x + 12000y)$

Subject to the constraints,

$9x + 12y \leq 180$ or $3x + 4y \leq 60$

$x + 3y \leq 30$

$x \geq 0, y \geq 0$

$3x + 4y \leq 60$

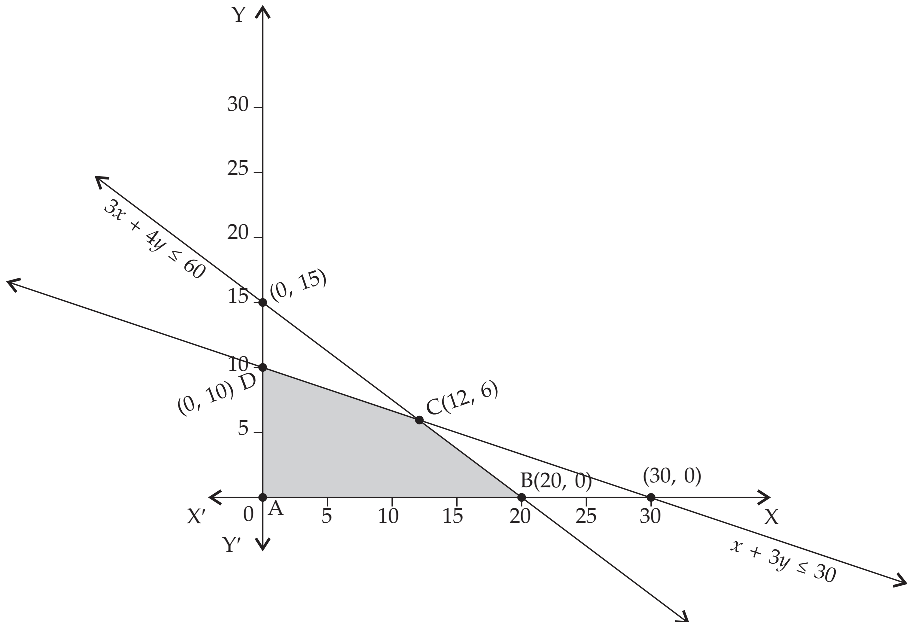
Let $3x + 4y = 60$

x	0	20	12
y	15	0	6

$x + 3y \leq 30$

Let $x + 3y = 30$

x	0	30	12
y	10	0	6



Corner Points	**P = ₹(8000*x* + 12000*y*)**
A(0, 0)	0
B(20, 0)	160000
C(12, 6)	96000 + 72000 = **168000**
D(0, 10)	120000

← **Maximum**

∴ **$x = 12$ and $y = 6$**

The maximum profit = ₹ 1,68,000

Or

$4x + y \geq 80$;

Let $4x + y = 80$;

x	0	20	15	2
y	80	0	20	72

$x + 5y \geq 115$,

$x + 5y = 115$;

x	0	115	40
y	23	0	15

$3x + 2y \leq 150$

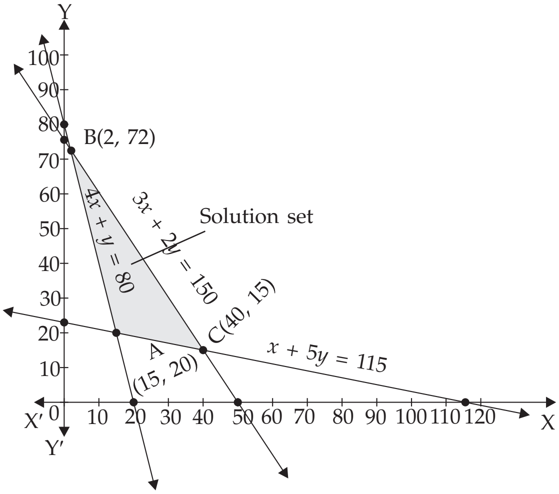
$3x + 2y = 150$

x	0	50	40
y	75	0	15

Corner points	$z = 6x + 3y$
A(15, 20)	90 + 60 = **150 (Minimum)**
B(2, 72)	12 + 216 = 228
C(40, 15)	240 + 45 = 285

Minimum value of **Z** is **150** at A (15, 20)

i.e., $x = 15, y = 20$.



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